

# The Iceberg Observation Problem: Using Multiple Agents to Monitor and Observe Ablating Target Sources

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**Abstract**—Ships that operate in polar regions continue to face the threat of floating ice sheets and icebergs generated from an ice ablation process. Systems have been implemented to track these threats, with varying degrees of success. In this paper, a definition is proposed for this tracking problem that re-casts it within a class of robotic, multiagent target observation problems. The focus in this new definition is on minimizing the time an initial contact for newly generated targets is obtained, as opposed to obtaining a contact long after a target has been generated. Focusing on the initial contact time provides for the ability to enact early countermeasures. A model is provided for the target sources, as well as metrics for computing costs associated with the model for reallocating robotic agents during an observation task. The effectiveness of the solution compared with an existing observation technique is demonstrated using simulation.

**Index Terms**—Field robotics, multiagent robotics, remote sensing

## I. INTRODUCTION

The danger of collision with floating ice is a threat that continues to exist for polar, ship-based operations [1]. Collision avoidance takes an amount of advance planning by the ships' crew, as a result of the slow speeds and lack of mobility of the craft. Therefore, a means of tracking floating ice is necessary for situational awareness. Usually, the information needed to build such a plan is derived from radar and visual observations. The International Ice Patrol (IIP), for example, provides data products for this purpose for maritime operations in the Newfoundland region [2].

Disadvantages exist for both radar and visual observation [3]. Smaller icebergs, for example, are difficult for a radar to track because of their low-magnitude radar cross section (RCS) with respect to sea clutter. In addition, satellite-based synthetic aperture radar (SAR) imaging is generally not available in real-time. Visual observation requires that a dedicated crew continuously monitors for new threats. Furthermore, ice threats are not completely observable in this manner, and once a threat is observed, depending on the type of ship, there could be a shortage of time for the crew of the ship to begin evasive maneuvers. Therefore, a method of remotely sensing these threats with a reduced emphasis on remote radar sensing may

be the best approach to the problem, with an additional focus on understanding the sources of these threats.

Icebergs are generated by an *ice ablation* process called *ice calving* [4]. Depending on temperature, glacial flow rate, and the base strength of their source glaciers, these ice masses have varying rates of generation. For example, Figure 1 is a diagram of the flow rates of Antarctic glaciers, which vary depending on the proximity of the glacier to the coast and other factors. This diagram shows that the flow rate is not the same at different interface regions of the ice with the ocean. The calving rate of ice masses would be higher at the regions with a greater flow rate. In addition, the current surface velocity of the ocean coupled with the generation rates dictates the spread of floating ice masses, which must be taken into account when determining a plan for addressing the issue. Flow rate information might not be immediately available to an observer; hence, observations of the calved ice masses would be a more appropriate means of determining which sources should have a greater monitoring emphasis.

Based on observations of a floating ice source and the masses calved from it, a model can be developed for the locations on the source where new ice masses are generated with highest probability. Using this information, sensor resources can be reallocated to place a greater focus on the sources that generate more targets. In this case, a multiagent, robotic observation system is proposed to act as the sensor system, based on an existing robotic sensing framework problem. The use of robotic agents provides for a flexible and less expensive solution to the iceberg observation problem. In addition, monitoring can be a continuous process and better coverage can be attained by the use of multiple autonomous agents. Similar methods are used in other marine robotics applications, such as determining a model of tidal currents for allowing multiple autonomous agents to navigate effectively [5].

The remainder of this paper is organized as follows: Section II defines the general problem of observing these ice masses, Section III outlines our methodology for addressing the problem, Section IV describes our model for target sources and how

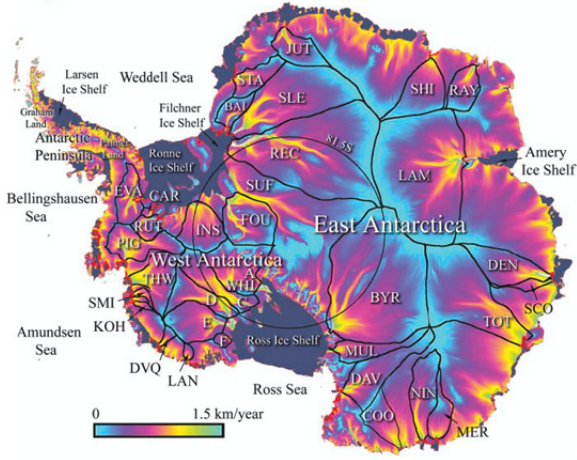


Fig. 1. Flow rates of Antarctic glaciers. Image courtesy of NASA.

it is generated, Section V presents simulation data and results for our algorithms, and Section VI concludes the paper.

## II. PROBLEM DEFINITION

### A. Overview

Since the masses of floating ice are moving targets, and an *observation region* can be defined for a set of agents with the ice masses floating into and out of the region, the problem of remotely observing these targets is very similar to a class of well-known, robotic observation problems defined as Cooperative Multi-robot Observation of Multiple Moving Targets (CMOMMT) [6]. The primary objective of solutions to this problem is the determination of the best placement of robots that will observe targets that move through the region of interest. This placement must maximize the amount of time that any given target is observed.

This problem class can be adapted through changes to the base assumptions, as described below, to fit many observation tasks; however, the general problem itself is difficult enough that many solutions have been developed by researchers [7]–[13]. In this case, the CMOMMT problem is useful as a framework that can be extended to obtain a general model for the iceberg observation problem.

### B. Definition

We define the standard CMOMMT problem using the following definitions:

$\mathcal{S}$ : A two-dimensional, polyhedral region.

$\mathcal{R}$ : A set of  $M$  robot positions  $r_i$ .

$\mathcal{O}(t)$ : A set of target positions  $o_j(t)$ , such that  $o_j(t)$  is contained within  $\mathcal{S}$  at time  $t$ . The cardinality of  $\mathcal{O}(t)$  at time  $t$  is  $N(t)$ .

The robots have omnidirectional sensors that are limited in observation range. The target positions contained in  $\mathcal{O}(t)$  can change, and the initial positions are not known in advance. The members of  $\mathcal{O}(t)$  are assumed to enter and exit  $\mathcal{S}$  through well-defined entrances on the boundary of  $\mathcal{S}$ . Figure 2 is an illustration of an example layout for this problem. With these

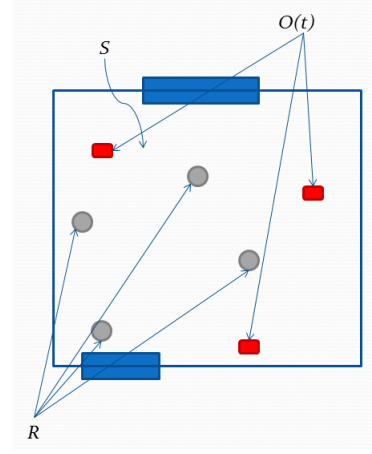


Fig. 2. An example layout of the objects defined in Section II-B. The thick bars at the top and bottom of  $\mathcal{S}$  are target entrances and exits.

definitions and assumptions, the  $M \times N(t)$  matrix  $A_{ij}(t)$  for a specific time  $t$  can be defined with the elements

$$a_{ij}(t) = \begin{cases} 1 & \text{if robot } r_i \text{ is monitoring target } o_j(t) \text{ in } \mathcal{S} \\ 0 & \text{otherwise.} \end{cases}$$

With this matrix, the CMOMMT objective function for time  $t$ , with a range 0 to  $T$ , divided into  $\Delta t$  steps, is defined to be<sup>1</sup>

$$\sum_{t=0}^T \sum_{j=1}^{N(t)} \bigvee_{i=1}^M a_{ij}(t). \quad (1)$$

The goal is to maximize this objective function; the time that each object is being monitored by at least one robot under some set of assumptions. Additional base assumptions exist for the CMOMMT problem; however, they are not relevant to the framework as given here. Further details are contained in the reference [6].

For the problem of ice mass generation and observation, the targets will be assumed to have a *highly variable* but *bounded* velocity, which is reasonable to assume for the effects of ocean currents. We will also define a *target source*: a target source is any member of  $\mathcal{O}(t)$  at  $t = 0$ .

The CMOMMT assumptions are modified as follows:

- 1) No specific target entrances or exits are defined at the boundary of  $\mathcal{S}$ . Targets may exit from any location on the boundary of  $\mathcal{S}$ .
- 2) Targets enter from target sources located within the interior of  $\mathcal{S}$ ; *i.e.*,  $\mathcal{O}(t)$  grows over time as the target sources undergo ablation.
- 3) *A priori* information regarding approximate target source locations is available at  $t = 0$ .

Assumptions 1 and 2 are derived directly from the ice mass problem as described in Section I. The assumption that the targets are generated from ablation indicates that an upper bound on the number of targets that a given source can generate in a time interval exists. Assumption 3 is necessary to provide

<sup>1</sup>The  $\bigvee$  operator indicates a logical “or” over all members in the set.

an initial allocation of the agents directed to observe the sources.

The objective of this modified problem is slightly different from the general CMOMMT problem. Since early detection is paramount, the amount of time required to obtain the *initial contact* on a target contained within  $S$  must be minimized. The contact time, denoted by  $T_s$ , is defined as the time at which a robot's sensors detect the target referenced from the start of the ablation process. The target contacts will be obtained by the agents using some type of search pattern [14] [15] to execute their search, and any further target tracking will be accomplished by agents dedicated to that task.

However, as a result of sensor uncertainties, the sensors used by the robots will not have a probability of detection equal to 1. Hence, the contact time  $T_s$  for a *single target* is the contact time of a "perfect" sensor plus zero-mean noise. The noise variance is dependent on the sensor errors. In addition,  $T_s$  is different for each of the targets contained in  $\mathcal{O}(t)$  for a given  $t$  as a result of the different robot positions  $r_i$  with respect to the targets  $o_j(t)$ . This dependency means that to obtain the expected value of the contact time for a given target source, the expectation must be taken over all of the targets.

The objective function for this problem may then be defined to be the following:

$$\min_{\mathcal{R}} E[T_s | \mathcal{O}(t)], \quad (2)$$

where  $E[\cdot]$  denotes the expected value operator.

### III. METHODOLOGY

With the problem defined in Section II, a methodology can be outlined for minimizing the objective function using multiple robotic agents. For areas of ablation activity on a target source, more sensing resources will be required to ensure that all targets are observed in a minimum amount of time. Therefore, more resources should be allocated to regions with a high probability of a new target being generated. As the target generation probability for a particular region decreases, agents should be reallocated in a more equal manner around a target source. This agent allocation scheme will not only minimize the initial contact time, but it will also reduce the uncertainty in quantifying the activity around a particular target source.

To determine the appropriate probability density, observations must be incorporated into a model. The observations are composed of the following elements: the position at which the target was first observed, the observation time, and which agent made the observation. The model incorporates these observations as well as the current number of observed targets across all agents and the *a priori* probability density of a new target being formed. In the case of the very first observations being made, the *a priori* observed position of the target source itself is used for initialization purposes.

On-line analysis of the resulting *a posteriori* density will indicate whether to undergo a reallocation of resources. A metric of variability will be developed to indicate when the allocation should change, and new search regions will be devised from the most active regions in the model. The level



Fig. 3. Example probabilistic model of an ablating source.

of activity will be computed by determining which regions in the model have the highest probability of generating a new target.

Note that certain aspects of this methodology have some similarities to the coverage problem. However, in the coverage problem, the region of interest is generally static and the problem of interest is determining the most efficient means of observing the entire region or some aspect of the entire region. What is outlined here resembles an *adaptive* coverage solution, since a model of the target sources is determined and the resources (*i.e.*, agents) and regions of coverage are adapted to fit that model.

### IV. TARGET SOURCE MODEL

#### A. Model Definition and Computation

A probabilistic model for the target source as described in Section III would allow for multiple modes and for quantifying the spread of the targets that have been generated. An example of such a model is shown in Figure 3. The glacier-sea interface is indicated by the jagged line on the left. There are two regions of high target generation probability; therefore, a reallocation of resources would either split the number of currently active agents between the two regions, or allocate more agents to the mode of the distribution with a greater spread (corresponding directly to a greater area).

With respect to the given requirements, a Gaussian mixture model (GMM) [16] is one appropriate way of modeling the regions of activity on a target source; the individual components in the mixture will be assumed to directly correspond to the regions of activity at the glacier-sea interface. Hence, the mixture will act as the *a posteriori* density described in Section III; the *a priori* density is derived from the initial parameters for the search conducted by the agents, which are set as part of a mission planning process. Each agent will maintain its own "picture" of the model. This allows for each agent to make its own decisions based on how it views the model and redundancy in the case that an agent is disabled.

GMMs are defined in terms of the means and covariances of their components; *i.e.*, the probability density function  $p(x)$  for a GMM is defined as

$$p(x) = \sum_{j=1}^k w_j \phi(x | \mu_j, \Sigma_j) \quad (3)$$

where each  $w_j$  is the component weight, and  $\mu_j$  and  $\Sigma_j$  are the component means and covariance matrices, respectively.

As previously stated, each component in the mixture will correspond to a region of ablation activity. The covariance matrix of each component will be used to derive the extents of a region that the agents must monitor. Since the component weights are a measure of the influence of a component, they will be used in ranking which component requires more observation resources in concert with the spread of the observation.

To generate the model, the agents must obtain *measurements* of target state. Before defining what a measurement contains, we will consider each target to have a state vector  $\mathbf{x}$  of the following form:

$$\mathbf{x} = [x \quad \dot{x} \quad y \quad \dot{y}]^T \quad (4)$$

where  $(x, y)$  are the target position components, and  $(\dot{x}, \dot{y})$  are the target velocity components.

Measurements of the target states must be shareable across agents, to provide for the generation of a complete picture of the target source model at each agent. Therefore, each measurement will be labeled with an *agent identifier*, *target identifier*, and the *time* at which the measurement was obtained. The remainder of the measurement vector consists of position observations, perturbed by noise. A complete measurement vector  $\mathbf{z}$  will be defined as

$$\mathbf{z} = [l \quad n \quad t \quad x_m \quad y_m]^T \quad (5)$$

where  $l$  is the label assigned to the target,  $n$  is the agent identifier,  $t$  is the observation time, and  $(x_m, y_m)$  is the noise-perturbed position observation.

Given a set of measurements of target position, the GMM components can be computed. *Expectation-maximization* [17] (EM) is used to obtain the GMM components once a sufficient number of measurements has been obtained. As measurements are collected, either through the actions of an individual agent or by obtaining measurements from other agents, the EM algorithm is run again on the new set of measurements, developing a clearer picture of the regions of activity for a particular target source. In this case, we are interested in the spread of the iceberg positions with respect to the target sources, so we use the  $(x_m, y_m)$  components of the measurement vector  $\mathbf{z}$  as the samples in the EM algorithm.

The EM algorithm is initialized by first running *k-means clustering* over the position measurements. The results of the clustering are used as the initial means in the EM algorithm, with the covariances initialized to identity matrices. After the algorithm has computed the corresponding model, the final association probabilities are used to assign measurements to components. This allows us to compute further metrics based on the model.

It should be noted that one requirement of the EM algorithm is that the number of mixture components is provided to the algorithm. Since part of the task we are attempting to accomplish is identifying the regions of activity, we will likely not have sufficient *a priori* information to make a good guess as to what the number of components should be. Hence, we

generate several models with a different number of mixture components. We then use the *corrected Akaike information criterion* [18] [19] (AICc) to select which of the models that we use. The AICc is defined as follows:

$$AICc = 2k - 2 \ln L + \frac{2k(k+1)}{n-k-1} \quad (6)$$

where  $k$  is the number of parameters in the model,  $L$  is the maximized value of the likelihood function of the mixture, and  $n$  is the number of measurements. The ideal model for a given set of measurements minimizes the loss of information provided by the measurements. That is, the model that minimizes the AICc is the model we use to perform resource allocation.

Once we have the target spread and the target source centers, we can further extend the model through other observable parameters to aid in resource allocation. For multiple observations of a target, the agent that observes the target will estimate the components of the target velocity. Unlike positional information, this information is *not* shared across agents when sharing measurements: the velocity is used as part of determining the cost of the agent moving from its current search region to a different search region.

## B. Metrics

To perform the allocation, we must first score the model and compute metrics. With these metrics, the most effective agent configuration can be determined. Each agent computes the following metrics based on the measurements and the model:

- Mean target contact time across all components
- Mean target contact time per component
- Average target velocity per component

Note that the first metric is a key portion of the objective function defined in Section II-B. The latter two metrics can be, as an example, combined into a cost function, coupled with agent parameters, which can then be used for agent assignment.

## V. SIMULATION

We compare the model generated by our multiple agent modeling algorithm with that of a more traditional scanning method for icebergs using airborne radar in a simulation environment. That is, a scan pattern composed of multiple parallel-transects (*i.e.*, a “lawnmower” pattern) across an area of interest by a single agent, similar to the scan pattern used in the study described in reference [20].

We implemented and simulated the necessary robot controllers and the target sources using the software libraries provided by the Player/Stage project [21]. The metrics recorded as a result of the simulation to determine algorithm performance include target coverage and contact time. Target coverage is important to consider as missed targets contribute to an incomplete model. We performed 50 trials for each case, with the target source configured as shown in Figure 4, with target velocities that correspond to ocean currents in the regions of interest. Each trial was time limited to approximately five minutes of collection time; the ablation rates of each activity

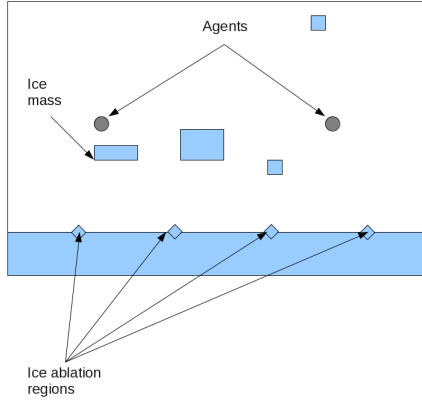


Fig. 4. Target source configuration.

TABLE I  
SUMMARY OF ACTIVITY REGION PARAMETERS.

Active Region	$\mu_{dims}$	$\sigma_{dims}$	$\mu_{vel}$	$\sigma_{vel}$
1	5 m x 5 m	0.5 m	(-0.5, 0.5) m/s	0.05 m/s
2	5 m x 5 m	0.5 m	(-0.5, 0.5) m/s	0.05 m/s
3	2 m x 2 m	1.0 m	(0, 0.5) m/s	0.05 m/s
4	5 m x 5 m	0.5 m	(0.5, 0.5) m/s	0.05 m/s

point were thus accelerated to accommodate this time limit. The sizes of the targets that are calved from the activity regions are also varied; the source parameters are summarized in Table I. Each parameter of the calved targets is modeled according to Gaussian errors, where  $(\mu_{dims}, \sigma_{dims})$  is the mean and standard deviation of the dimensions of each target, and  $(\mu_{vel}, \sigma_{vel})$  is the mean and standard deviation of the velocities. The mean sizes and velocities of the icebergs were chosen based on the frequency of classes of iceberg sizes and speeds [22]; smaller icebergs are more commonly encountered in practice. The ice mass from which the icebergs are calved was placed at the position  $(x, y) = (50 \text{ m}, -5 \text{ m})$ , with a length of 100 m, and a width of 10 m. Note that the width is the only “important” dimension for the ice mass; the length may be considered to measure the extent of the ice mass at the glacier-sea interface.

Each robot was equipped with a sensor that could detect a target within a circle that has a 10 m radius. For the multi-agent solution, each agent conducted its search using a parallel-transect pattern beginning at a starting point a distance away from the ablating source, within a rectangular cell. The agents began at points  $(x, y) = (18 \text{ m}, 5 \text{ m})$ ,  $(40 \text{ m}, 5 \text{ m})$ , and  $(68 \text{ m}, 5 \text{ m})$  respectively. The single scanning agent was placed at the initial position  $(x, y) = (0 \text{ m}, 5 \text{ m})$ . All agents had a minimum speed of 0.1 m/s and a maximum speed of 1 m/s.

The basic metrics from the results of the simulations are given in Table II. Average  $T_s$  is the average acquisition time of a target referenced from the time at which it is generated from a given activity point on an ablating source. Average model  $T_s$  is the mean acquisition time from the start of a given agent’s mission; in this case, these times are referenced

TABLE II  
SIMULATION METRICS SUMMARY.

Scenario	Pct. Coverage	Avg. $T_s$	Avg. Model $T_s$
Single Agent	21%	19 s	89 s
2 Agents	80%	6.5 s	58 s
3 Agents	97%	7.1 s	70 s

from the first agent. From these results and for this particular scenario, for near 100% coverage, three agents is sufficient, although the performance in terms of acquisition time suffers. However, given that the targets themselves are not fast-moving, the loss in acquisition time, when compared, is not a significant issue for the multiagent cases.

In the case of the models generated, we desire a resulting Gaussian mixture model that exhibits the following characteristics:

- Sufficient target containment. *I.e.*, all targets, while perhaps not measured, can be accounted for within the model.
- Good component separation. Good separation allows for generation of search regions that maximize target coverage for a single agent, and minimize the chances of agent collision. Separation also indicates that individual activity areas were sufficiently identified.

Figure 5 shows a selection of the mixture models generated by each of the scenarios given above. Target measurements are denoted by stars; the target source is the gray rectangle, with activity points identified by the circles overlaid on the source. The Gaussian mixtures are represented by the heatmaps, with their means represented by the black crosses.

Compared with the other results, the model generated by the single agent solution is not sufficient for this scenario. While there is good component separation and given that the measurements are all clustered above a single source, there is insufficient target containment when all targets are taken into account, although an activity region is identified. In the multiagent cases, target containment is not an issue; search regions can be generated that can provide sufficient coverage for all of the activity points on the ice mass. Component separation is sufficient, providing for the generation of distinct search regions. However, when generating search regions from the models, there may be a slight overlap between the regions assigned to particular agents. Additionally, for the model shown in Figure 5(c), one of the components may need to be inflated to provide proper coverage for a given region.

Overall, when taking the metrics into consideration with the resulting model, using multiple agents is the superior solution for model generation. This is true for target coverage, acquisition time, and model containment. However, depending on the scenario, it is important to consider the number of agents that are actually required. In the case here, while using three agents attained nearly 100% target coverage, the model that was generated as shown in Figure 5(b) (two agents) provides sufficient target containment. However, component separation and potentially resources are sacrificed, as a result of the size of



the search regions that would be generated. Therefore, a future task is to determine how to predict the appropriate number of agents based on the model that was generated.

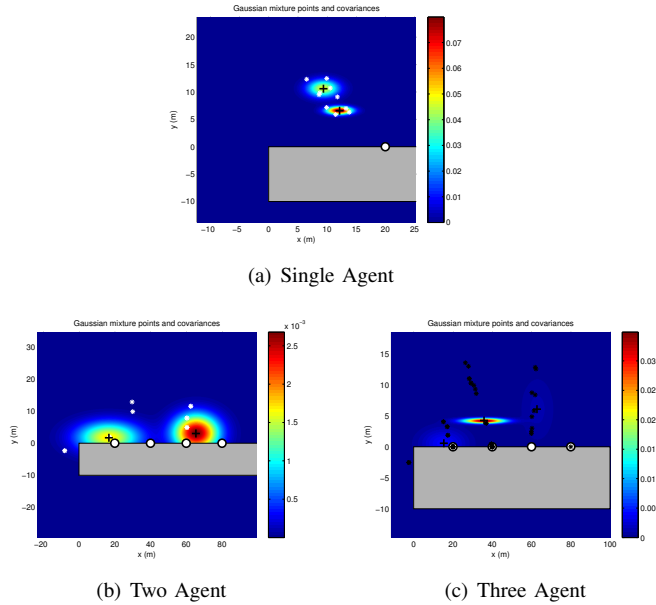


Fig. 5. Gaussian mixture models created from each of the scanning solutions. The red areas (component center) indicate highest probabilities of target presence.

## VI. CONCLUSION

A problem definition corresponding to the real-world problem of observing ice masses that are calved from glaciers has been provided. The approach to the definition is based on the existing framework of the CMOMMT problem. We modify the CMOMMT assumptions and add additional assumptions as part of this new definition. The new assumptions that are key to this new definition are the assumptions regarding placement of the target sources within the region and the generation of targets through ablation. An objective function has been defined for this problem: it is desired to minimize the time required to obtain the initial contact on a newly generated target.

As part of addressing this observation problem, we have outlined a methodology for developing a probabilistic model for predicting the locations where a target will be generated, using information from initial contacts of generated targets. Metrics can be computed using that model and the measurements used to generate it to determine the costs associated with performing resource allocation. We compared our distributed modeling methods to a model generated using a more traditional scanning method.

Future work includes incorporating the model into a true resource allocation algorithm and moving agents from search region to search region based on the results computed from the model, and implementing the algorithms on real robot hardware.

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